

# PRESSURE-IMPULSE DIAGRAM OF A BEAM UNDER EXPLOSION

## Influence of the indirectly affected part

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## INTRODUCTION

The robustness of a structures can be defined as its ability to exhibit an appropriate behaviour in case of exceptional event such as terrorist attack, impact, explosion... and, in particular, to avoid the development of a progressive collapse in case of local failure.

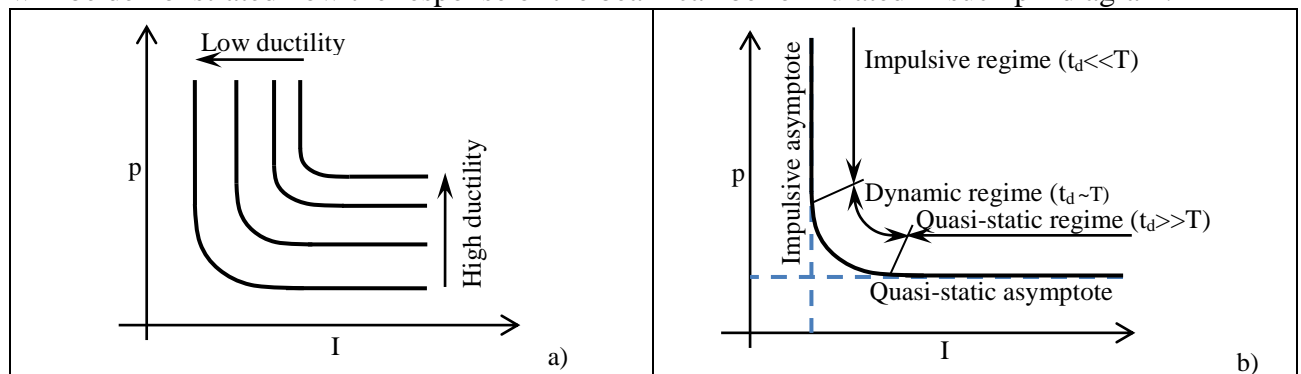
If reference is made to recent norms and standards for construction structures, it is clearly specified that the robustness of structures should be ensured; however, no clear guidance is given on how to achieve an appropriate level of robustness for a specific construction.

Accordingly, different research activities are under development all around the world in this field and the proposal of easy-to-apply design recommendations aimed at ensuring an appropriate robustness to construction has been identified as one of the main priorities in the field of civil engineering.

This article, as a contribution to this research thematic, focuses on the structural behaviour of a beam, which is formally extracted from a frame system, subjected to blast loading. The specificity of this study concerns the consideration of the lateral restraint and the inertia offered by the rest of the frame, referred to as the *indirectly affected part*. The dynamic structural behaviour of this beam is studied in large displacements and moderate rotations, assuming a kinematic failure given by the classical plastic flexural mechanism. Lateral torsional instability is assumed to be prevented. The goal is to determine the required ductility of the beam subjected to a triangular blast loading.

## 1 PROBLEM FORMULATION

Within the present study, it is demonstrated how the behaviour of a beam subjected to a blast loading can be expressed through a pressure-impulse (p-I) diagram, which is a standard tool in blast engineering. Indeed, the use of such diagrams is preferred since they indicate, with a simple reading, the required ductility of a given structural element according to the pressure and impulse (p,I) delivered by the blast loading [1] as illustrated in *Fig. 1.a*. Such a curve is detailed in *Fig. 1.b* for a specific ductility of a structural element. As illustrated in *Fig. 1.b*, one key parameter defining the regime of the structural element in case of blast loading is the ratio of the characteristic period of the structure  $T$  to the positive phase duration of the blast load  $t_d$ . In particular, three regimes can be identified, i.e. the impulsive, the quasi-static and dynamic regimes. Within the present paper, it will be demonstrated how the response of the beam can be formulated in such p-I diagram.



*Fig. 1:* (a) Schematic of a p-I diagram and (b) regimes identified in one curve of the p-I diagram.

## 1.1 Description of the problem

The considered problem consists in the structural dynamic analysis of a beam, subjected to a uniformly distributed loading  $p(t)$  applied along the length of the beam  $2l$ . The structure is characterized by the mass of the floor  $M_s$ , the equivalent stiffness of the beam  $K_s$ , the elastic stiffness of the lateral restraint  $K^*$  and the lumped mass of the rest of the structure  $M^*$ .

The load pulse is idealized as a triangular pulse (Fig. 2) described by:

$$p(t) = p_0 \left( 1 - \frac{t}{t_d} \right); I = p_0 \frac{t_d}{2} (2l) \quad (1)$$

where  $t_d$  and  $p_0$  are the positive phase duration and the peak blast loading,  $t$  is the time and  $I$  is the impulse delivered to the beam.

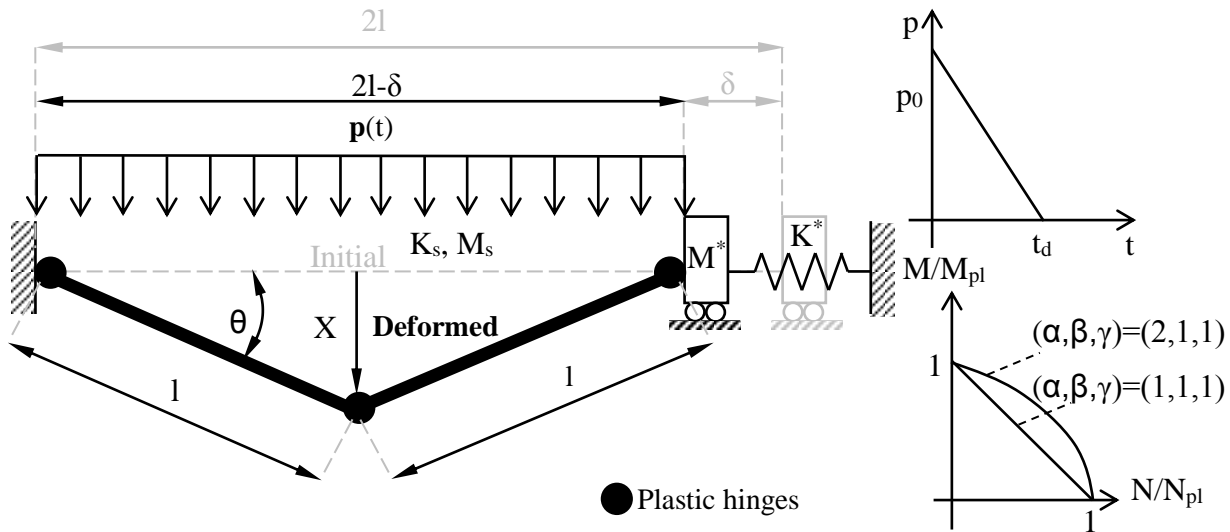


Fig. 2: Investigated structure and idealized blast load.

Further to explosion, the plastic mechanism is formed and followed by the development of membrane forces into the beam. A generalised plastic interaction  $M$ - $N$  law between the bending moment  $M$  and the axial force  $N$  at the plastic hinge level is introduced in the model:

$$\left( \frac{M}{M_{pl}} \right)^\beta + \gamma \left( \frac{N}{N_{pl}} \right)^\alpha = 1 \quad (2)$$

where  $M_{pl}$  and  $N_{pl}$  are the plastic bending and axial resistance of the beam. Symbols  $\alpha, \beta, \gamma$  depend on the material and the geometry of the beam. Some values are provided for steel, concrete and composite structural elements in [2]–[5].

The main assumptions adopted are: the lateral restraint remains elastic, the beam-to-column joints are perfectly rigid and fully resistant, the axial elongation of the plastic hinges under bending moment and membrane forces and the elastic elongation of the beam are negligible and the material law of the beam is elastic perfectly plastic.

## 1.2 Governing equations

Assuming that the deformed configuration of the beam is affine to the plastic mechanism, the displacement field is fully described by the displacement at mid-span  $X$ . Therefore, the beam is modelled as a single degree-of-freedom (SDOF) structure. Equivalent blast loading  $F_{ext}$ , internal forces, including the deformation energy in the beam  $F_{int,1}$  and in the lateral restraint  $F_{int,2}$ , as well as inertial forces are obtained by projection of the continuous fields in the assumed shape. The resulting equation of motion reads

$$\left( M_s + 4M^* \frac{X^2}{l^2} \right) \ddot{X} + 4M^* \frac{X\dot{X}^2}{l^2} + F_{int,1} \left( K_s X, M \left( N \left( X, \dot{X}, \ddot{X} \right) \right) \right) + F_{int,2} (X) = F_{ext}(t) \quad (3)$$

The membrane force  $N$  is obtained by writing the horizontal equilibrium equation at the end of the beam

$$N = \frac{2}{l} \left[ K^* \frac{X^2}{2} + M_{pl} \frac{X}{l} + M^* (\dot{X}^2 + X\ddot{X}) \left( 1 + \frac{1}{2} \left( \frac{X}{l} \right)^2 \right) \right]. \quad (4)$$

### 1.3 Dimensionless parameters and orders of magnitude

Introducing reference scales, in time  $T = \sqrt{M_s / K_s}$  and in length  $X_y$ , which correspond respectively to the characteristic period of the elastic beam and the displacement at yield, the set of equations (2)-(4) can be written in a dimensionless format where the dimensionless displacement and time are respectively  $\tilde{X} = X / X_y$  and  $\tau = t / T$ ,

$$\left( 1 + \eta \theta_y^2 \tilde{X}^2 \right) \ddot{\tilde{X}} + \eta \theta_y^2 \tilde{X} \dot{\tilde{X}}^2 + \tilde{F}_{int,1} \left( \tilde{X}, m \left( n \left( \tilde{X}, \dot{\tilde{X}}, \ddot{\tilde{X}} \right) \right) \right) + \tilde{F}_{int,2} (\tilde{X}) = \tilde{F}_{ext}(\tau) \quad (5)$$

$$m^\beta + \gamma n^\alpha = 1 \quad (6)$$

$$n = 4\xi \theta_y \tilde{X} + 8\xi \theta_y \psi_K \tilde{X}^2 + 4\psi_M \xi \theta_y \left( \dot{\tilde{X}}^2 + \tilde{X} \ddot{\tilde{X}} \right) \left( 1 + \frac{1}{2} \theta_y^2 \tilde{X}^2 \right) \quad (7)$$

where  $m = M / M_{pl}$  and  $n = N / N_{pl}$  are the dimensionless bending moment and axial force,  $\tilde{F}_{int,i} = F_{int,i} / R_m$  ( $i=1,2$ ) is the dimensionless internal force ( $R_m = 4M_{pl} / l$  is the flexural plastic resistance of the frame beam) and  $\tilde{F}_{ext}(\tau) = \tilde{p}(1 - \tau / \tau_d)$  is the dimensionless blast loading.

The set of equations (5)-(7) is solved with a numerical algorithm such as the nonlinear Newmark algorithm [6] in order to obtain the maximum response, i.e. the demand of ductility of the beam.

The demand of ductility  $\mu = \max(\tilde{X})$  depends on the following dimensionless parameters:

- $\psi_M = 4M^* / M_s$ , the ratio of the lateral mass to the mass of the beam ;
- $\psi_K = K^* / K_s$ , the ratio of the stiffness of the lateral restraint to the equivalent flexural stiffness of the beam ;
- $\xi = (M_{pl} / 2l) / N_{pl}$ , the ratio of bending to axial strengths ;
- $\theta_y = X_y / l$ , the yield rotation ;
- $\tilde{p} = p_0 l / R_m$ , the peak overpressure of the blast loading ;
- $\tau_d = t_d / T$  (or  $\tilde{I} = \tilde{p} \tau_d / 2$  which is the impulse delivered to the beam).

For the protection of staff and equipment through the attenuation of blast pressure and to shield them from the effects of fragments and failing portions of the structure, recommended deformation limits are given under category 1 in *Table 1*. For the protection of structural elements themselves from collapse under the action of blast loading, the recommended deformation limits are given under protection category 2 in *Table 1* [7].

The dimensionless parameter  $\psi_K$  depends on the structural elements and their configuration. To cover a wide range of realistic cases (see examples in *Fig. 3*), the dimensionless parameter  $\psi_K$  is assumed to vary from 0 to 2. The parameter  $\psi_M$  depends on the distribution of mass in the structure, and is assumed to vary from 0 to 5. These parameters are obtained by performing a static condensation of the mass and stiffness matrices of the structure.

Table 1. Maximum values of rotation  $\theta$  and ductility  $\mu$  for steel structural elements according to two levels of protection defined by the US Army. [7]

	Protection category 1		Protection category 2	
	$\theta$	$\mu$	$\theta$	$\mu$
Structural steel beams and plates	$2^\circ=35 \text{ mrad}$	10	$12^\circ=210\text{mrad}$	20

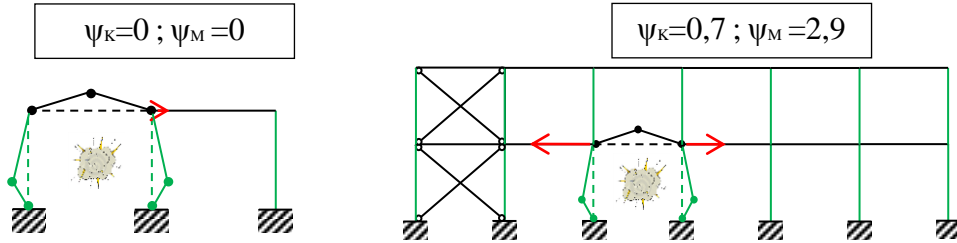


Fig. 3: Values of the dimensionless parameters  $\psi_K$  and  $\psi_M$  for different steel structure configurations with IPE270 beams (5 m), HEA240 columns (4,5 m), CHS175x5 braces and a linear mass of the floor equals to 2500 kg/m.

The dimensionless parameters  $\xi$  and  $\theta_y$  depend only on the properties of the profile and its span.

Table 2 shows the orders of magnitude of the parameters  $\xi$  and  $\theta_y$  for different values of the ratio of the span  $2l$  to the beam depth  $h$ . These numbers are obtained with some class-1 S355 steel-grade steel profiles (such as I, H-shaped or tubular profiles).

Table 2. Minimum and maximum values of the dimensionless parameters  $\xi$  [%] and  $\theta_y$  [mrad] for steel beams with S355 steel grade according to different ratios  $2l/h$  (obtained from Arcelor Mittal catalogue).

$2l/h$	10	20	30
$\min(\xi)$ [%]	3,3	1,7	1,1
$\max(\xi)$ [%]	4,2	2,1	1,4
$\min(\theta_y)$ [mrad]	4,7	9,3	14
$\max(\theta_y)$ [mrad]	5,5	11	16,5

Three regimes exist depending on  $\tau_d$ , i.e. the shortness of the duration of the loading compared to the natural period of the beam. For impulsive ( $\tau_d \ll 1$ ) and quasi-static ( $\tau_d \gg 1$ ) regimes, analytical asymptotic solutions are obtained by means of simple energy conservation. In the intermediate regime however ( $\tau_d \approx 1$ ), the set of equations (5)-(7) must be solved numerically.

## 2 NUMERICAL SOLUTIONS

### 2.1 Illustrative example

Consider a structure composed by a steel beam IPE 270 with a S355 steel grade and a length  $2l$  of 5,4m. The linear mass of the reinforced concrete floor is equal to 2500kg/m. According to Villette's formulae [4], the coefficients  $\alpha, \beta$  and  $\gamma$  are respectively equal to 2,1 and 1 for bending about the strong axis. The peak overpressure and the positive phase duration are respectively equal to 320kN/m and 0,1s. The characteristic displacement, force and time are respectively  $X_y = 0,026m$ ,  $R_m = 255kN$  and  $T = 0,02s$ . They scale the results shown in Fig. 4.

The dimensionless numbers of this problem are

$$\psi_K = 1; \psi_M = 1; \xi = 2 \% ; \theta_y = 10^{-2} \text{ rad} ; \tilde{p} = 3,4; \tau_d = 5; \tilde{I} = 8,5 \quad (8)$$

Fig. 4-a illustrates the dimensionless displacement versus the dimensionless time. Fig. 4-b shows the evolution of the dimensionless internal forces according to the dimensionless displacement. Four points labelled A, B, C and D indicate the different stages of the response.

Firstly, at point A, the plastic mechanism of the beam has just been formed, meaning that  $\tilde{X} = 1$ . The sum of the dimensionless internal forces is close to 1 since the effect of the lateral restraint is negligible. At point B, the maximum dimensionless displacement rises up to 20, a bit after the moment  $\tau_d$  where the blast loading stops. Between points A and B, the membrane force decreases the moment in plastic hinges and therefore, the equivalent internal force in the beam. However, the internal force in the lateral restraint increases to reach a value 50% over the plastic resistance of the beam. After reaching the maximum displacement, the beam is subjected to an elastic loading in the opposite direction. Indeed, the lateral restraint returns a part of its elastically stored energy to the beam. At point C, the plastic mechanism is developed in the opposite direction. Finally, at point D, the beam starts vibrating indefinitely elastically.

This detailed response corresponds to only one point in a (p-I) diagram, namely a required ductility of 20 for the couple  $(p; I) = (3, 4; 8, 5)$ . This point is represented by a green dot in Fig. 5-a. This approach can be repeated for different couple  $(p; I)$  to obtain a curve in the (p-I) diagram.

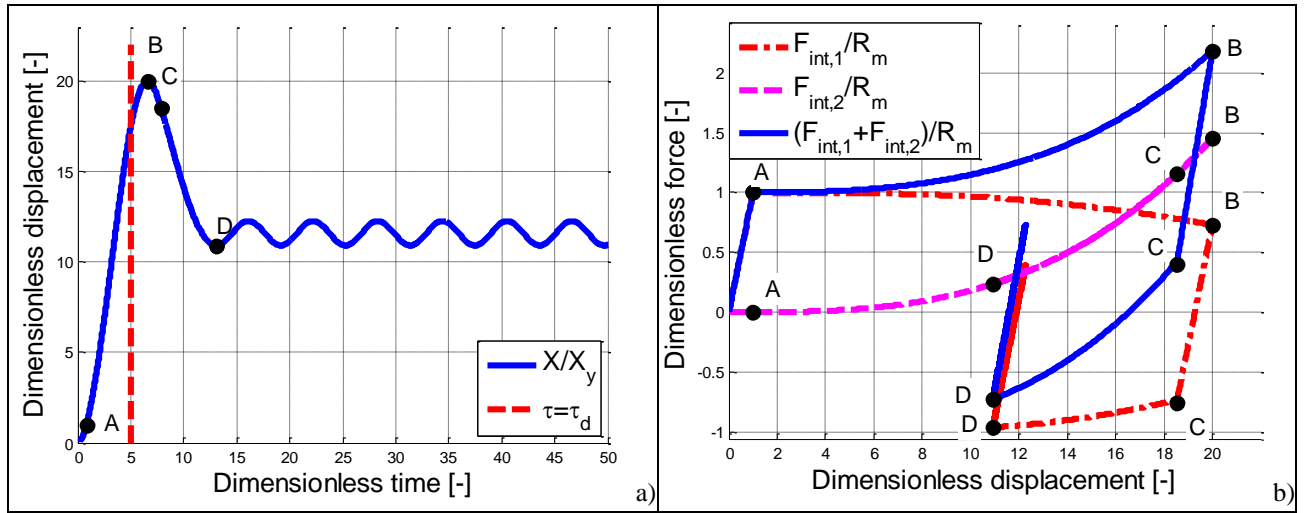


Fig. 4: (a) Displacement versus time and (b) internal forces versus displacement for a given example ( $\psi_K = 1; \psi_M = 1; \xi = 2\%$ ;  $\theta_y = 10^{-2} rad$ ;  $\tilde{p} = 3, 4; \tau_d = 5; \tilde{I} = 8, 5$ ).

## 2.2 P-I diagrams

P-I diagrams are represented for  $\psi_K = 1$  (Fig. 5-a) and  $\psi_K = 0$  (Fig. 5-b), while other parameters are chosen as  $\xi = 2\%$ ,  $\theta_y = 10^{-2} rad$ ,  $\psi_M = 1$ . Each curve represents the required ductility. Solid lines represent the asymptotes of the curves, which correspond to quasi-static and impulsive loadings and could be obtained analytically by writing the energy conservation principle. The good agreement with the numerical results serves as a validation of the numerical code. For a given blast loading, the ductility is seen to decrease from 26,2 to 20 when the lateral restraint is considered.

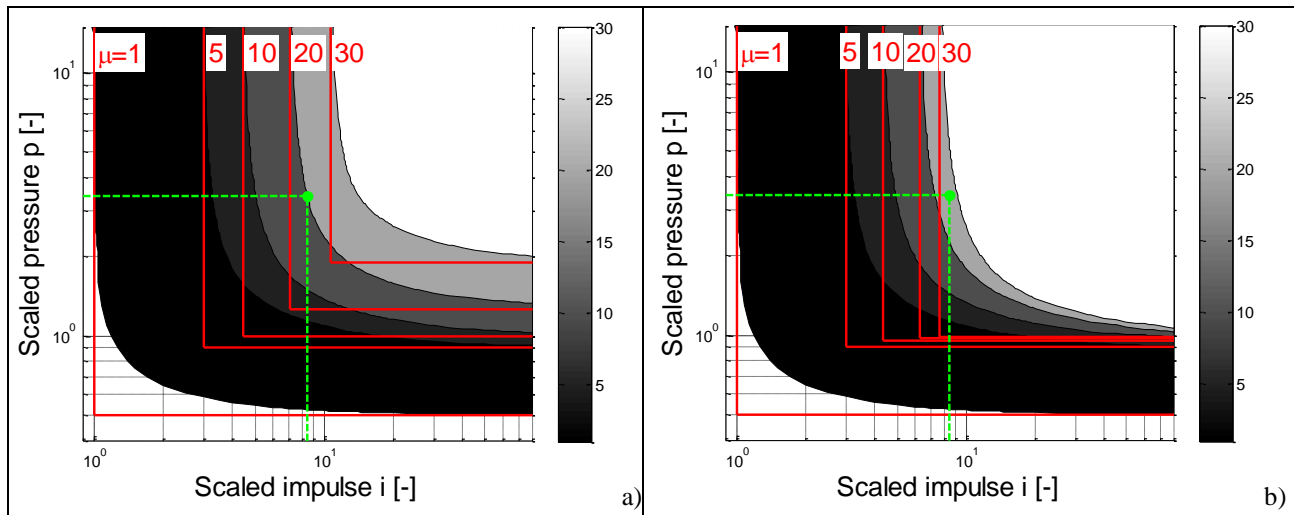


Fig. 5: Normalized p-I diagrams in logarithmic axes for  $\xi = 2\%$ ,  $\theta_y = 10^{-2} \text{ rad}$ ,  $\psi_M = 1$  (a)  $\psi_K = 1$  and (b)  $\psi_K = 0$ .

### 3 SUMMARY

The present paper presents how the response of a beam subjected to a blast loading can be formulated using a p-I diagram, including the effect of the structures around the considered beam called the indirectly affected part. As a result of the so-realised dimensionless analysis, four structural dimensionless parameters significantly affecting the demand in terms of ductility of the beam are identified. Two of them are linked to properties of the indirectly affected part (i.e. its lateral stiffness and its equivalent mass). Another one is related to the mechanical properties of the investigated beam (i.e. its flexural and axial resistances). The last parameter is linked to the cinematic of the problem (i.e. the rotation of the beam at its extremities when the plastic mechanism is developed). Within the present paper, it has been shown how one of these parameters affect the beam response. A next step of the presented study will be to establish an analytical expression to derive the curves of the p-I diagrams in order to avoid the numerical resolution of the non-linear dynamic equations which is still required for the time being.

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**KEYWORDS:** pressure-impulse diagram, robustness, frame beam, blast loading, membrane force.

### **ABSTRACT**

The robustness of a structures can be defined as its ability to exhibit an appropriate behaviour in case of exceptional event such as terrorist attack, impact, explosion... and, in particular, to avoid the development of a progressive collapse in case of local failure.

If reference is made to recent norms and standards for construction structures, it is clearly specified that the robustness of structures should be ensured; however, no clear guidance is given on how to achieve an appropriate level of robustness for a specific construction.

Accordingly, different research activities are under development all around the world in this field and the proposal of easy-to-apply design recommendations aimed at ensuring an appropriate robustness to construction has been identified as one of the main priorities in the field of civil engineering.

The presented paper, as a contribution to this research field, introduces a study investigating the structural response of a beam subjected to a close-field local internal blast loading.

In the literature, the pressure-impulse (p-I) diagram is commonly used to design elements or structures for a given blast loading [1]. The p-I diagram is a spectrum representing the level sets of required ductility  $\mu$  (defined as the ratio between the plastic and elastic deformation) for a given structural system (*Fig. 1*). Indeed, the use of such diagrams is preferred since they indicate, with a simple reading, the required ductility of a given structural element according to the pressure and impulse (p,I) delivered by the blast loading [1] as illustrated in *Fig. 1* in which three regimes can be identified: the impulsive, the quasi-static and dynamic regimes.

If the blast loading is slower than the response time of the structural element, the iso-damage curve tends to a quasi-static asymptote. However, if it is the opposite, the curve tends to the impulsive asymptote. Between these two extreme cases, the structural element is under a dynamic regime.

Considering the structural system under investigation, i.e. a structural beam subjected to a blast loading, recent works have indicated the crucial need to account for the membrane effects taking place in case of extreme loading applied to frame beams [2], [3], membrane effects which are strongly affected by the properties of the structure around the considered beam (called *the indirectly affected part* [2]) and by the (M-N) interaction developing at the plastic hinge level. The effect of the so-defined indirectly affected part is represented by a spring with a stiffness  $K^*$  and by a mass  $M^*$  in the model representing the investigated beam as illustrated in *Fig. 2-b*.

### **CONCLUSIONS**

Within the presented study, it is demonstrated how the behaviour of a beam subjected to a blast loading can be expressed through a pressure-impulse (p-I) diagram, taking into account the surrounding structure (*Fig. 2*). In particular, through the realised dimensionless analysis, four structural dimensionless parameters significantly affecting the demand in terms of ductility of the beam are identified. Two of them are linked to properties of the indirectly affected part (i.e. its lateral stiffness  $K^*$  and its equivalent mass  $M^*$ ). Another one is related to the mechanical properties of the investigated beam (i.e. its flexural and axial resistances). The last parameter is linked to the

cinematic of the problem (i.e. the rotation of the beam at its extremities when the plastic mechanism is developed). Within the present paper, it is shown how one of these parameters affect the beam response.

A next step of the presented study will be to establish an analytical expression to derive the curves of the p-I diagrams in order to avoid the numerical resolution of the non-linear dynamic equations which is still required for the time being.

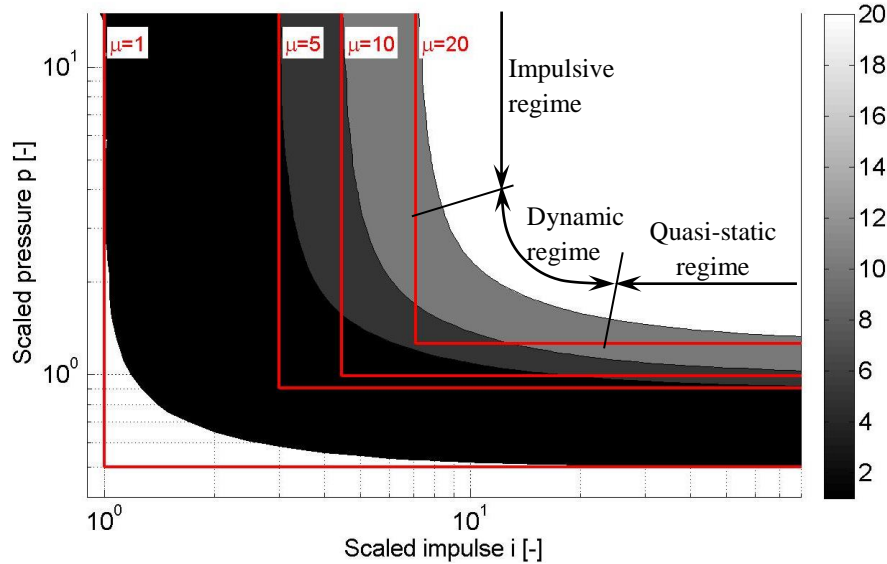


Fig. 1: Dimensionless pressure-impulse (p-I) diagram of a non-linear beam subjected to blast loading (in logarithmic axes).

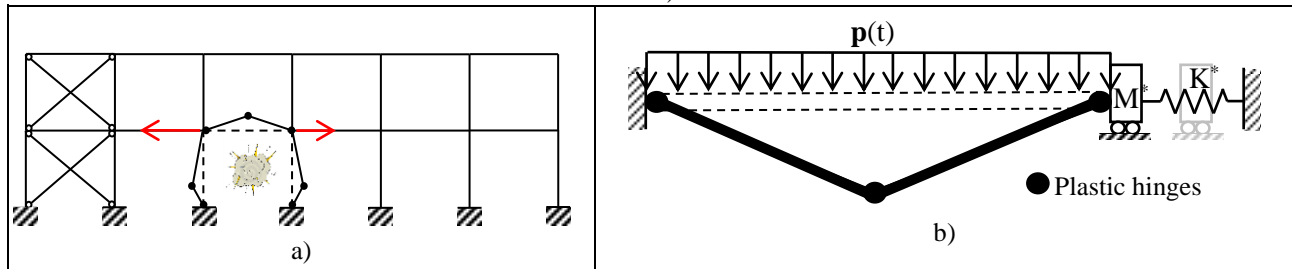


Fig. 2. a) Complete structure; b) Extraction of the substructure.

## ACKNOWLEDGMENT

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